

ON THE MODELLING OF THE FORCES FROM A 8R CARDAN TRANSMISSION

Alina BORCA*, Dorin DIACONESCU*, Radu SĂULESCU*

* Transilvania University of Braşov, Dept. of Product Design and Robotics,
dvdiaconescu@unitbv.ro

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Abstract: A new eccentric cardan transmission of 8R type, destined to transmit the power between two parallel shafts with adjustable distance between them, is modelled in this paper. Using only revolute joints, this transmission eliminates the intermediate telescopic joint (which is characteristic for the classic 2-cardan transmissions) and can transmit higher torques by using the cardan joints with eccentric axes. The paper presents the movements and the forces/torques modelling of this transmission and some features, which result from the numerical simulations.

1. INTRODUCTION

For the cardan transmissions, which are used to transmit the power between two parallel shafts with adjustable distance between them, a modern optimization direction relies on the utilization of the eccentricity as mounting possibility of bearings with high carrying capacity. Therefore, this paper presents movements and forces/torques modelling of a new eccentric homokinematical cardan transmission, of 8R type, illustrated in Fig. 1 (the transmission is homokinematical, because it is symmetrical; that's why the two cardan joints are connected *orthophased*; it means that in the initial position, when the joint's axis A belongs to the plan formed by the extremal shafts' axes, the joint's axis E is perpendicular on this plan; see Fig. 1.b.). Because the transmission must have 1 DOF and must be symmetrical, it is centred with a symmetrization kinematical chain of type R-S/P-R-S/P-R, which consists of the elements 3, 4, 5 and 8 (Fig. 1).

2. ON THE KINEMATICAL MODELLING

According to Fig.1, the kinematical modelling is based on the utilization of the homogeneous operators [2]:

$$T_{1' \rightarrow 0} \cdot T_{1 \rightarrow 1'} \cdot T_{2 \rightarrow 1} \cdot T_{3 \rightarrow 2} \cdot T_{4 \rightarrow 3} \cdot T_{5 \rightarrow 4} \cdot T_{6 \rightarrow 5} = T_{7' \rightarrow 0} \cdot T_{7 \rightarrow 7'} \cdot T_{6 \rightarrow 7}, \quad (1)$$

where $T_{n \rightarrow m}$ is the passing matrix from the trihedron $x_n y_n z_n$ to the trihedron $x_m y_m z_m$; for example:

$$T_{1 \rightarrow 0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad T_{7 \rightarrow 7'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c\varphi_7 & -s\varphi_7 \\ 0 & 0 & s\varphi_7 & c\varphi_7 \end{bmatrix}; \quad (2),(3)$$

A system with 12 scalar equations and 7 unknown sizes ($\theta_{21}, \theta_{32}, \theta_{43}, \theta_{54}, \theta_{65}, \theta_{76}, \varphi_7$) results after the mathematical processing. For the solving simplification of this equations system, the next kinematical equalities are taken into consideration: $\varphi_1 = \varphi_7$, $\theta_{21} = \theta_{76}$, $\theta_{32} = \theta_{65}$, $\theta_{43} = \theta_{54}$; these equalities result from the transmission symmetry. Based of this simplification, the independent equations and their solutions are obtained by means of the soft *Maple V*.

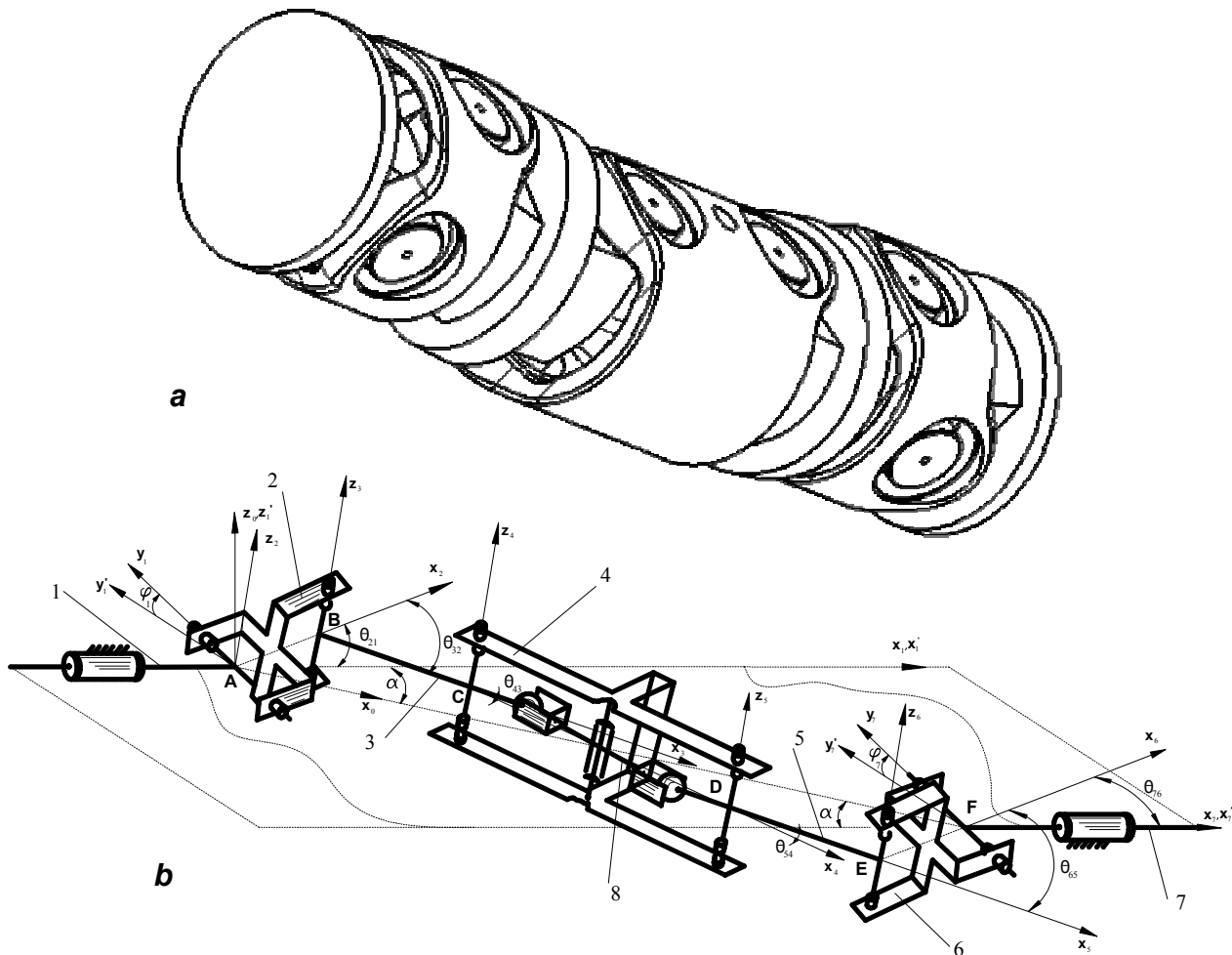


Fig. 1 Schemes and notations of the orthophased eccentric cardan mechanism of 8R type

The obtained results are used in the forces/torques modelling; for exemplification, in Table 1 there are systematized the variations of the displacements from the transmission's joints, for the following numerical data: $\alpha = 5^\circ$; 20° and $AF = 1500$; 1800 mm (see Fig. 1).

3. THE ANALYTICAL MODELLING OF THE FORCES/TORQUES

For the transmission's forces/torques modelling, beside the notations from the Fig. 1, b, there are used the following notations and premises:

$2r$ — the distance from contact zones of cardan forks (see Fig.1);
 M_1, M_2 — *module* of the input and respectively the output torque of the transmission;
 $Qx_ny_nz_n$ — reference trihedron which is solidary to the element (n), where $Q = A, B, C, \dots$;
 T_{Qn}, B_{Qn} — *module* of the torsion and respectively bending torque from the joint Q ($Q = A, B, C, \dots$), which are described in the trihedron of the element (n); the next element, of the joint Q , has the same torques, but in the opposite direction (Fig.2,3 and 4);
 A_{Qn}, R_{Qn}, H_{Qn} — *module* of the axial force, the radial force and the normal force from the joint Q ($Q = A, B, C, \dots$), which are described in the trihedron of the element (n); the next element, of the joint Q , has the same forces, but in the opposite direction.

The modelling of the transmission's forces and torques relies on the following premises:

- Friction losses, from the revolute joints, are neglected;
- The torsional inertial effects are neglected, because the rotation speed is small;
- The transmission's elements are rigid bodies;
- The output torque M_2 is considered as an independent parameter.

The main objective of this modelling is the determination of the dependence between the input torque M_1 and the output torque M_2 and respectively determination of the dependences between the joints' reactions ($T_{Qn}, B_{Qn}, A_{Qn}, R_{Qn}, H_{Qn}; Q = A, B, C, \dots$) and output torque.

With this purpose, in the Fig.2, 3 and 4 there are represented the reactions of the isolated elements 2, 3, 4 and 8.

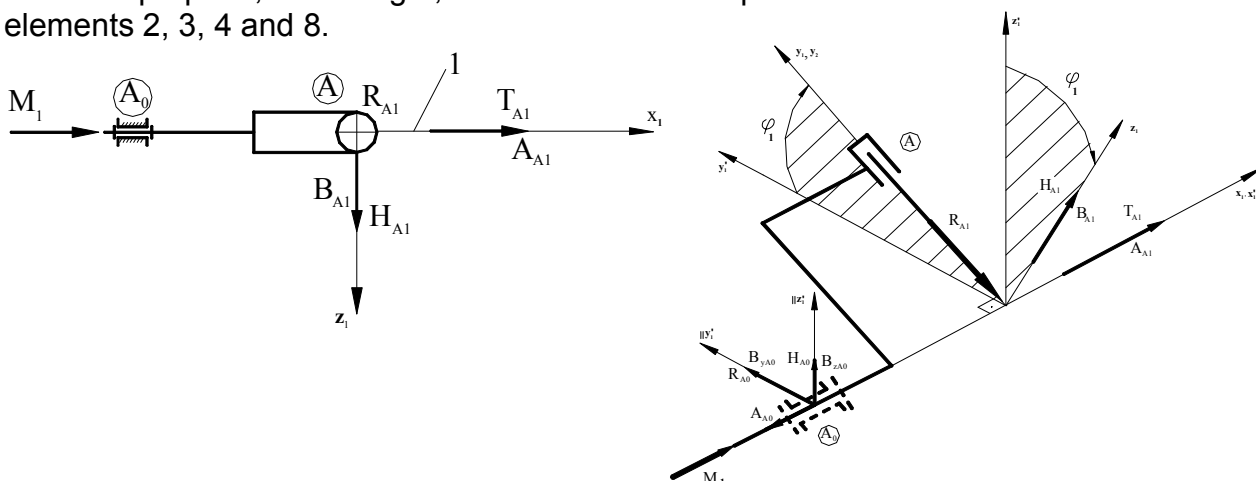


Fig. 2 Reactions from the joints of the element 1

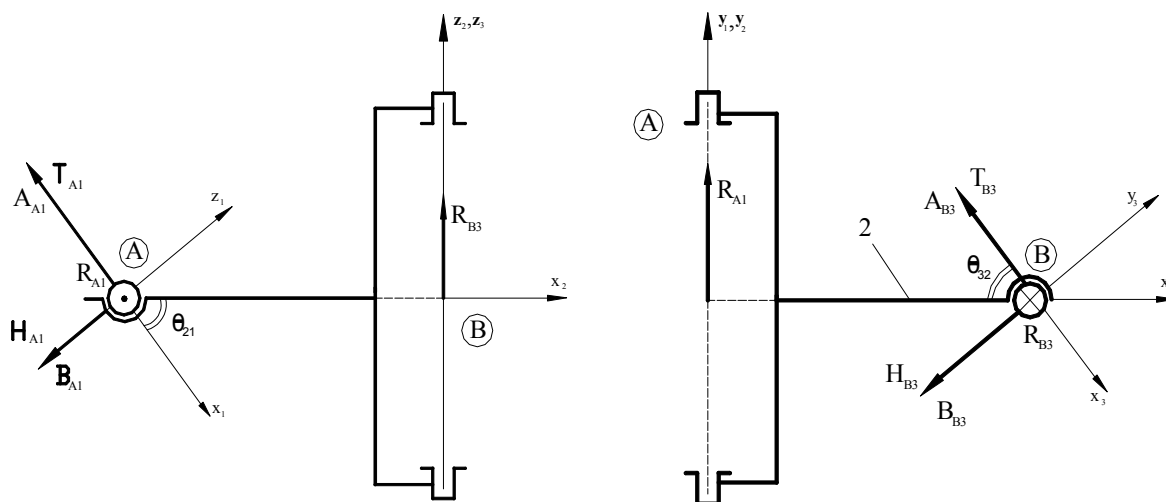


Fig. 3 Reactions from the joints of the element 2

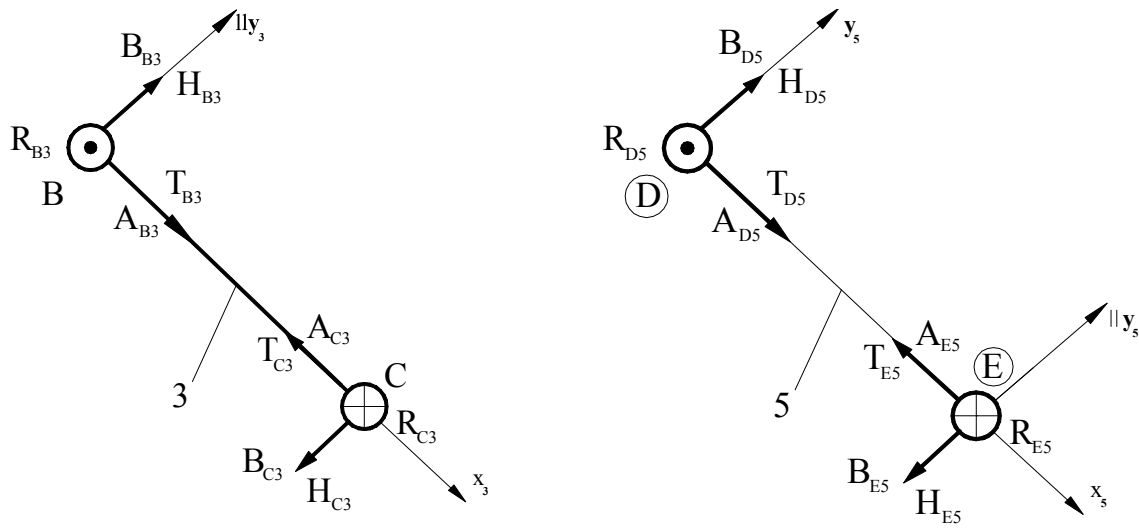


Fig. 4 Reactions from the joints of the elements 3 and 5

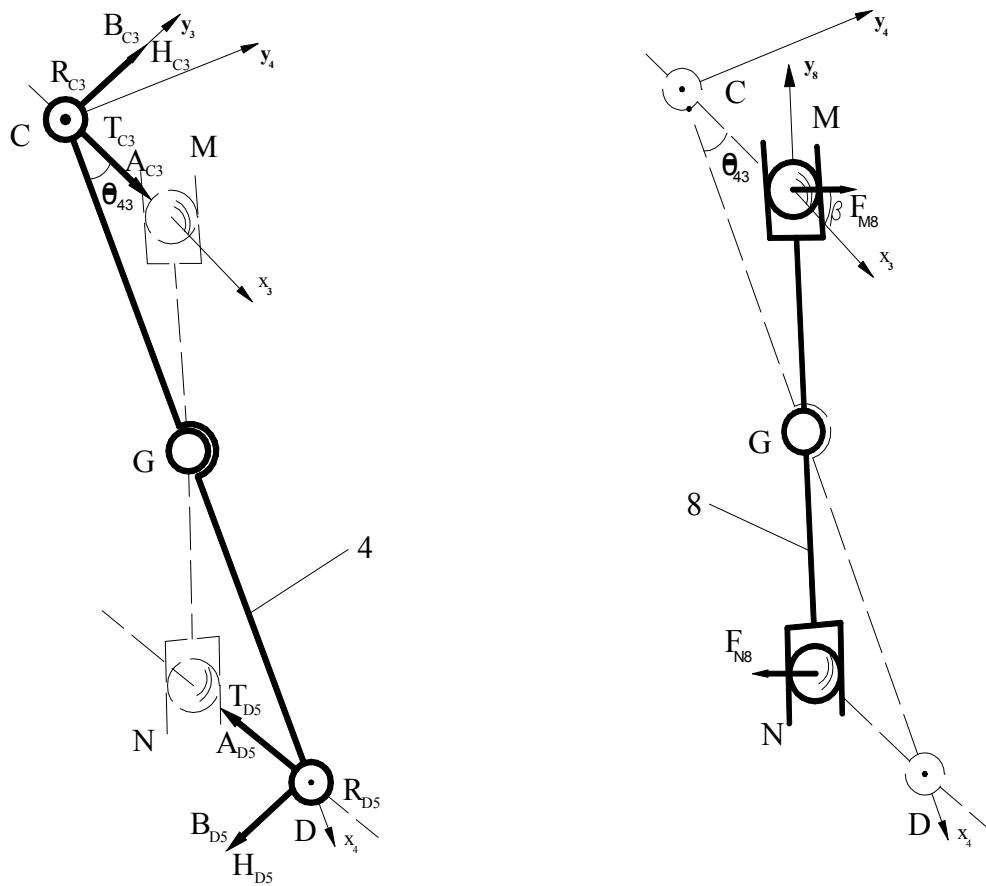


Fig. 5 Reactions from the joints of the central shaft 4 and of the central arm 8

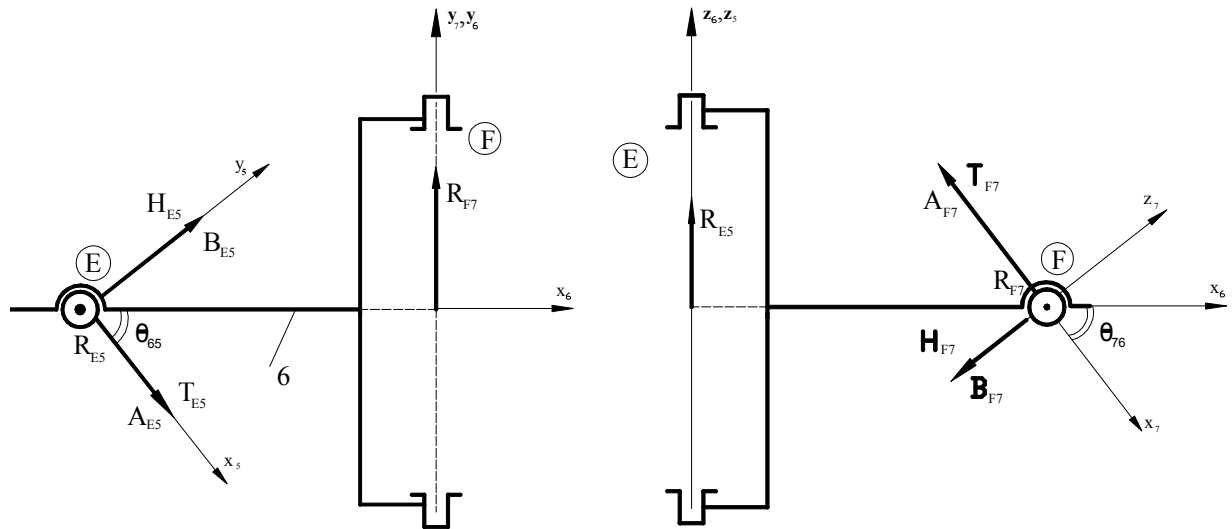


Fig. 6 Reactions from the joints of the element 6

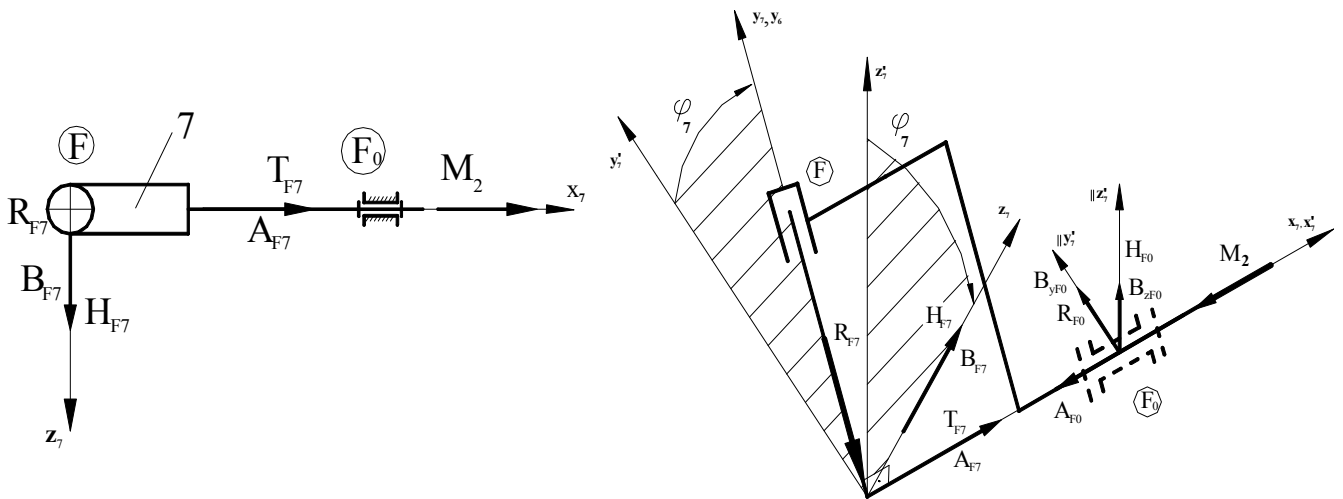


Fig. 7 Reactions from the joints of the element 7

According to Fig. 2, in the system $Ax_1'y_1'z_1'$, can be written the following correlations between the forces and the torques which load the element 1:

$$A_{A_1} - A_{A_0} = 0 \quad (4)$$

$$-R_{A_1} \cdot \cos \varphi_1 + R_{A_0} - H_{A_1} \cdot \sin \varphi_1 = 0 \quad (5)$$

$$-R_{A_1} \cdot \sin \varphi_1 + H_{A_0} + H_{A_1} \cdot \cos \varphi_1 = 0 \quad (6)$$

$$T_{A_1} + M_1 = 0 \quad (7)$$

$$-B_{A_1} \cdot \sin \varphi_1 + H_{A_0} \cdot AA_0 + B_{yA_0} = 0 \quad (8)$$

$$B_{A_1} \cdot \cos \varphi_1 - R_{A_0} \cdot AA_0 + B_{zA_0} = 0 \quad (9)$$

According to Fig. 3, in the trihedron $Ax_2y_2z_2$, can be written the following correlations between the forces and the torques, which load the element 2:

$$-A_{A_1} \cdot \cos \theta_{21} - H_{A_1} \cdot \sin \theta_{21} - A_{B_3} \cdot \cos \theta_{32} - H_{B_3} \cdot \sin \theta_{32} = 0 \quad (10)$$

$$R_{A_1} + A_{B_3} \cdot \sin \theta_{32} - H_{B_3} \cdot \cos \theta_{32} = 0 \quad (11)$$

$$-H_{A_1} \cdot \cos \theta_{21} + R_{B_3} + A_{A_1} \cdot \sin \theta_{21} = 0 \quad (12)$$

$$-T_{A_1} \cdot \cos \theta_{21} - B_{A_1} \cdot \sin \theta_{21} - T_{B_3} \cdot \cos \theta_{32} - B_{B_3} \cdot \sin \theta_{32} = 0 \quad (13)$$

$$T_{B_3} \cdot \sin \theta_{32} - B_{B_3} \cdot \cos \theta_{32} + R_{B_3} \cdot AB = 0 \quad (14)$$

$$A_{B_3} \cdot \sin \theta_{32} \cdot AB - H_{B_3} \cdot \cos \theta_{32} \cdot AB - B_{A_1} \cdot \cos \theta_{21} + T_{A_1} \cdot \sin \theta_{21} = 0 \quad (15)$$

According to Fig. 4, in the trihedrons $Bx_3y_3z_3$ and $Dx_5y_5z_5$, can be written the following correlations between the forces and the torques, which load the elements 3 and 5:

$$A_{B_3} - A_{C_3} = 0 \quad A_{D_5} - A_{E_5} = 0 \quad (16, 17)$$

$$H_{B_3} - H_{C_3} = 0 \quad H_{D_5} - H_{E_5} = 0 \quad (18, 19)$$

$$R_{B_3} - R_{C_3} = 0 \quad R_{D_5} - R_{E_5} = 0 \quad (20, 21)$$

$$T_{B_3} - T_{C_3} = 0 \quad T_{D_5} - T_{E_5} = 0 \quad (22, 23)$$

$$B_{B_3} - B_{C_3} + BC \cdot R_{C_3} = 0 \quad B_{D_5} - B_{E_5} + DE \cdot R_{E_5} = 0 \quad (24, 25)$$

$$H_{C_3} \cdot BC = 0 \quad H_{E_5} \cdot DE = 0 \quad (26, 27)$$

According to Fig. 5, in the trihedron $Cx_4y_4z_4$ and in the trihedron $Gx_8y_8z_8$, can be written the following correlations between the forces and the torques which load the central shaft 4 and the symmetrical central arm 8:

$$A_{C_3} \cdot \cos \theta_{43} - H_{C_3} \cdot \sin \theta_{43} - A_{D_5} \cdot \cos \theta_{43} + H_{D_5} \cdot \sin \theta_{43} = 0 \quad (28)$$

$$A_{C_3} \cdot \sin \theta_{43} + H_{C_3} \cdot \cos \theta_{43} + A_{D_5} \cdot \sin \theta_{43} - H_{D_5} \cdot \cos \theta_{43} = 0 \quad (29)$$

$$R_{C_3} - R_{D_5} = 0 \quad (30)$$

$$T_{C_3} \cdot \cos \theta_{43} - B_{C_3} \cdot \sin \theta_{43} - T_{D_5} \cdot \cos \theta_{43} + B_{D_5} \cdot \sin \theta_{43} = 0 \quad (31)$$

$$T_{C_3} \cdot \sin \theta_{43} + B_{C_3} \cdot \cos \theta_{43} - T_{D_5} \cdot \sin \theta_{43} - B_{D_5} \cdot \cos \theta_{43} + CD \cdot R_{D_5} = 0 \quad (32)$$

$$H_{D_4} \cdot CD = 0 \quad \Rightarrow \quad (H_{D_5} \cdot \cos \theta_{43} + A_{D_5} \cdot \sin \theta_{43}) \cdot CD = 0 \quad (33)$$

$$F_{M_8} = F_{N_8} \quad (34)$$

According to Fig. 6, in the trihedron $Fx_6y_6z_6$, can be written the following correlations between the forces and the torques, which load the element 6:

$$-A_{F_7} \cdot \cos \theta_{76} - H_{F_7} \cdot \sin \theta_{76} + A_{E_5} \cdot \cos \theta_{65} + H_{E_5} \cdot \sin \theta_{65} = 0 \quad (35)$$

$$R_{F_7} - A_{E_5} \cdot \sin \theta_{65} + H_{E_5} \cdot \cos \theta_{65} = 0 \quad (36)$$

$$-H_{F_7} \cdot \cos \theta_{76} + R_{E_5} + A_{F_7} \cdot \sin \theta_{76} = 0 \quad (37)$$

$$-T_{F_7} \cdot \cos \theta_{76} - B_{F_7} \cdot \sin \theta_{76} + T_{E_5} \cdot \cos \theta_{65} + B_{E_5} \cdot \sin \theta_{65} = 0 \quad (38)$$

$$-T_{E_5} \cdot \sin \theta_{65} + B_{E_5} \cdot \cos \theta_{65} + R_{E_5} \cdot EF = 0 \quad (39)$$

$$-A_{E_5} \cdot \sin \theta_{65} \cdot EF + H_{E_5} \cdot \cos \theta_{65} \cdot EF - B_{F_7} \cdot \cos \theta_{76} + T_{F_7} \cdot \sin \theta_{76} = 0 \quad (40)$$

According to Fig. 7, in the trihedron $Fx_7y_7z_7$, can be written the following correlations between the forces and the torques, which load the element 7:

$$A_{F_7} - A_{F_0} = 0 \quad (41)$$

$$-R_{F_7} \cdot \cos \varphi_7 + R_{F_0} - H_{F_7} \cdot \sin \varphi_7 = 0 \quad (42)$$

$$-R_{F_7} \cdot \sin \varphi_7 + H_{F_0} + H_{F_7} \cdot \cos \varphi_7 = 0 \quad (43)$$

$$T_{F_7} - M_2 = 0 \quad (44)$$

$$-B_{F_7} \cdot \sin \varphi_7 - H_{F_0} \cdot FF_0 + B_{yF_0} = 0 \quad (45)$$

$$B_{F_7} \cdot \cos \varphi_7 + R_{F_0} \cdot FF_0 + B_{zF_0} = 0 \quad (46)$$

By means of the *Maple V* software, the following solutions result from this equations' system:

$$A_{Qn} = 0; \quad R_{Qn} = 0; \quad H_{Qn} = 0; \quad (47)$$

$$M_1 = M_2; \quad F_{N_8} = F_{M_8} \quad (48, 49)$$

$$B_{yA_0} = -B_{yF_0} = -M_2 \cdot \operatorname{tg} \theta_{21} \cdot \sin \varphi_1 \quad B_{zA_0} = -B_{zF_0} = M_2 \cdot \operatorname{tg} \theta_{21} \cdot \cos \varphi_1 \quad (50, 51)$$

$$T_{A_1} = -T_{F_7} = -M_2 \quad B_{A_1} = -B_{F_7} = -M_2 \cdot \operatorname{tg} \theta_{21} \quad (52, 53)$$

$$T_{B_3} = T_{C_3} = T_{D_5} = T_{E_5} = M_2 \cdot \frac{\cos \theta_{32}}{\cos \theta_{21}} \quad B_{B_3} = B_{C_3} = B_{D_5} = B_{E_5} = M_2 \cdot \frac{\sin \theta_{32}}{\cos \theta_{21}} \quad (54, 55)$$

$$T_{C_4} = T_{D_4} = T_{C_3} \cdot \cos \theta_{43} - B_{C_3} \cdot \sin \theta_{43}; \quad B_{C_4} = B_{D_4} = T_{C_3} \cdot \sin \theta_{43} + B_{C_3} \cdot \cos \theta_{43} \quad (56, 57)$$

4. NUMERICAL SIMULATIONS

The variations of the displacements from the transmission joints, which interfere in the determinations of the torsion and bending torques, are systematized in the Table 1.

$\alpha = 5^\circ$ și $AF=1500$ mm				$\alpha = 20^\circ$ și $AF=1500$ mm				$\alpha = 5^\circ$ și $AF=1800$ mm				$\alpha = 20^\circ$ și $AF=1800$ mm			
φ_1	θ_{21}	θ_{32}	θ_{43}	θ_{21}	θ_{32}	θ_{43}		θ_{21}	θ_{32}	θ_{43}		θ_{21}	θ_{32}	θ_{43}	
0	0	-32.642	79.842	0	-12.90537	77.41745		0	-10.19234	33.11382		0	10.43958	27.45684	
15	1.2972	-32.86	79.853	5.38152	-13.8594	77.59667	1.297169	-10.40859	33.13741	5.38152	9.381451	27.9088			
30	2.5048	-33.499	79.882	10.3141	-16.62308	78.08171	2.504769	-11.04002	33.20173	10.3141	6.35914	29.10109			
45	3.54	-34.512	79.923	14.43276	-20.92505	78.73382	3.540025	-12.03673	33.28935	14.43276	1.766061	30.64029			
60	4.3329	-35.826	79.964	17.49524	-26.37812	79.37444	4.332874	-13.32169	33.37669	17.49524	-3.890792	32.0904			
75	4.8304	-37.346	79.993	19.37006	-32.54127	79.83647	4.830449	-14.79866	33.44044	19.37006	-10.09276	33.10269			
90	5	-38.967	80.004	20	-38.96712	80.00417	5	-16.36107	33.46374	20	-16.36107	33.46374			
105	4.8304	-40.578	79.993	19.37006	-45.23141	79.83647	4.830449	-17.90073	33.44044	19.37006	-22.2768	33.10269			
120	4.3329	-42.069	79.964	17.49524	-50.94938	79.37444	4.332874	-19.31544	33.37669	17.49524	-27.49013	32.0904			
135	3.54	-43.344	79.923	14.43276	-55.785	78.73382	3.540025	-20.51511	33.28935	14.43276	-31.73055	30.64029			
150	2.5048	-44.317	79.882	10.3141	-59.45823	78.08171	2.504769	-21.42626	33.20173	10.3141	-34.81992	29.10109			
165	1.2972	-44.928	79.853	5.38152	-61.75412	77.59667	1.297169	-21.99487	33.13741	5.38152	-36.67727	27.9088			
180	6E-16	-45.136	79.842	2.55E-15	-62.53526	77.41745	6.14E-16	-22.18808	33.11382	2.55E-15	-37.29376	27.45684			
195	-1.2972	-44.928	79.853	-5.38152	-61.75412	77.59667	-1.297169	-21.99487	33.13741	-5.38152	-36.67727	27.9088			
210	-2.5048	-44.317	79.882	-10.3141	-59.45823	78.08171	-2.504769	-21.42626	33.20173	-10.3141	-34.81992	29.10109			
225	-3.54	-43.344	79.923	-14.43276	-55.785	78.73382	-3.540025	-20.51511	33.28935	-14.43276	-31.73055	30.64029			
240	-4.3329	-42.069	79.964	-17.49524	-50.94938	79.37444	-4.332874	-19.31544	33.37669	-17.49524	-27.49013	32.0904			
255	-4.8304	-40.578	79.993	-19.37006	-45.23141	79.83647	-4.830449	-17.90073	33.44044	-19.37006	-22.2768	33.10269			
270	-5	-38.967	80.004	-20	-38.96712	80.00417	-5	-16.36107	33.46374	-20	-16.36107	33.46374			
285	-4.8304	-37.346	79.993	-19.37006	-32.54127	79.83647	-4.830449	-14.79866	33.44044	-19.37006	-10.09276	33.10269			
300	-4.3329	-35.826	79.964	-17.49524	-26.37812	79.37444	-4.332874	-13.32169	33.37669	-17.49524	-3.890792	32.0904			
315	-3.54	-34.512	79.923	-14.43276	-20.92505	78.73382	-3.540025	-12.03673	33.28935	-14.43276	1.766061	30.64029			
330	-2.5048	-33.499	79.882	-10.3141	-16.62308	78.08171	-2.504769	-11.04002	33.20173	-10.3141	6.35914	29.10109			
345	-1.2972	-32.86	79.853	-5.38152	-13.8594	77.59667	-1.297169	-10.40859	33.13741	-5.38152	9.381451	27.9088			
360	-1E-15	-32.642	79.842	-5.11E-15	-12.90537	77.41745	-1.23E-15	-10.19234	33.11382	-5.11E-15	10.43958	27.45684			

By means of the previous results and of the *Excel* software, there were presented the variations of *reduced* torsion torques T_{B3}/M_2 and T_{C4}/M_2 , in respect to φ_1 , and the variations of *reduced* bending moments B_{A1}/M_2 , B_{B3}/M_2 and B_{C4}/M_2 , in the same conditions.

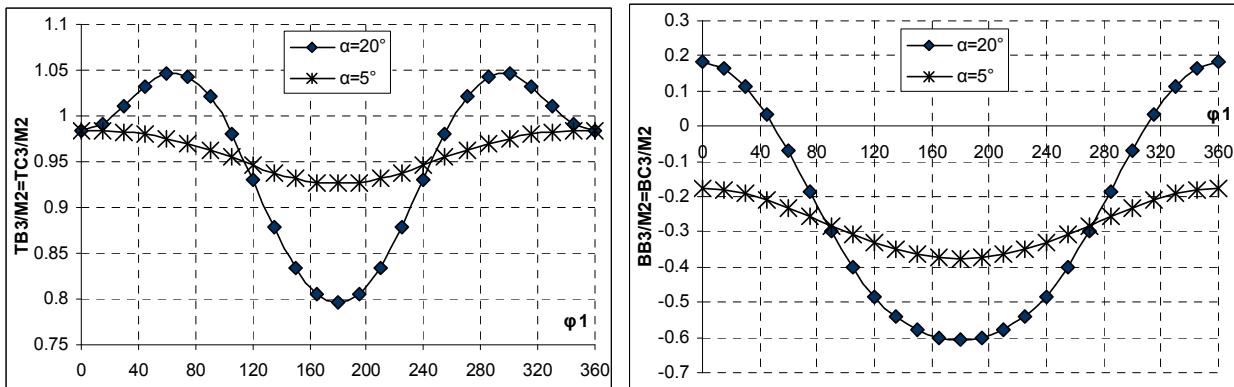


Fig. 8 Variations of the reduced torsion torque $T_{B3}/M_2 = T_{C3}/M_2$, and of the bending moment $B_{B3}/M_2 = B_{C3}/M_2$ for $AF=1800$ [mm], in respect to φ_1 [deg]

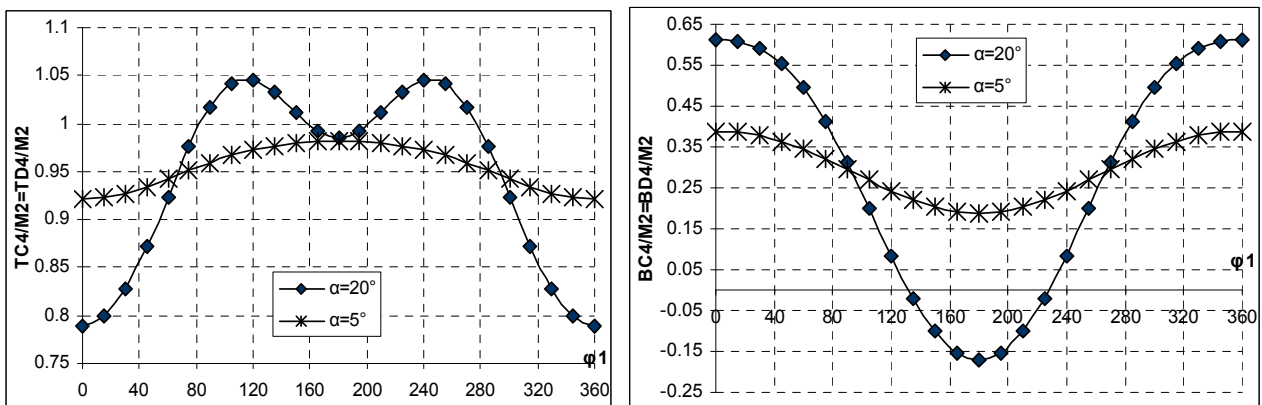


Fig. 9 Variations of the reduced torsion torque $T_{C4}/M_2 = T_{D4}/M_2$, and of the bending moment $B_{C4}/M_2 = B_{D4}/M_2$ for $AF=1800$ [mm], in respect to φ_1 [deg]

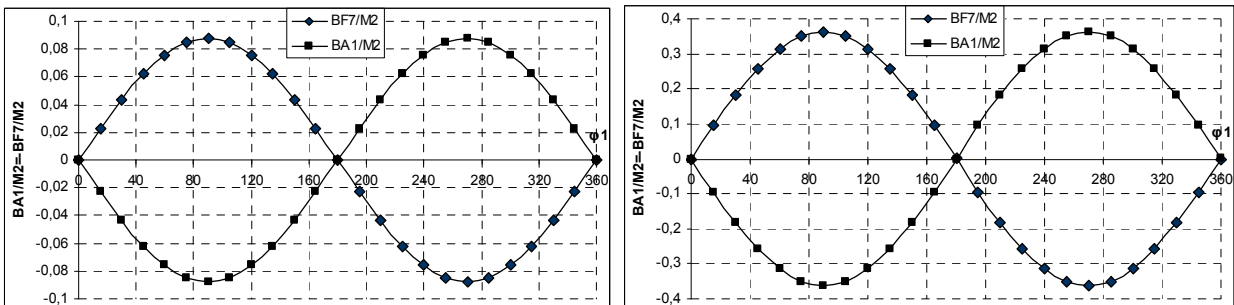


Fig.10 Variations of the reduced bending moment B_{A1}/M_2 for $\alpha = 5^\circ; 20^\circ$ and $AF=1800$ [mm], in respect to φ_1 [deg]

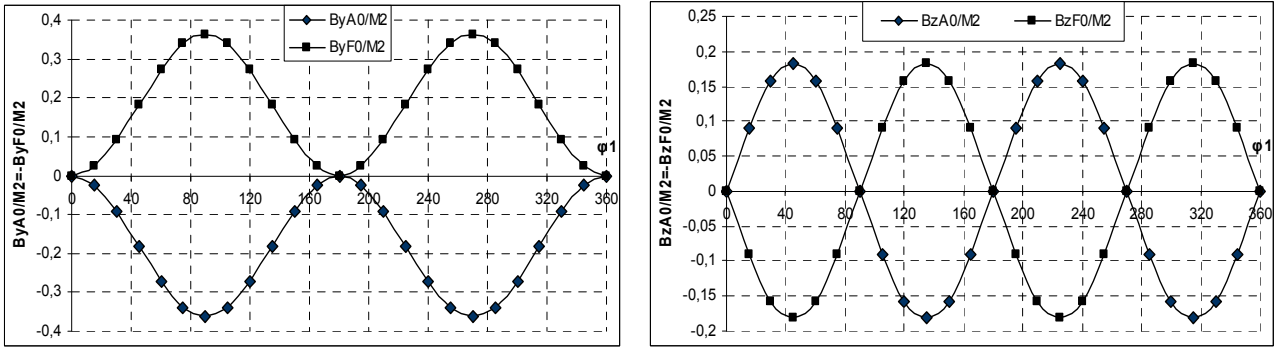


Fig.11 Variations of the reduced bending moments B_{yA0}/M_2 and B_{zA0}/M_2 for $\alpha = 20^\circ$ and $AF=1800$ [mm], in respect to φ_1 [deg]

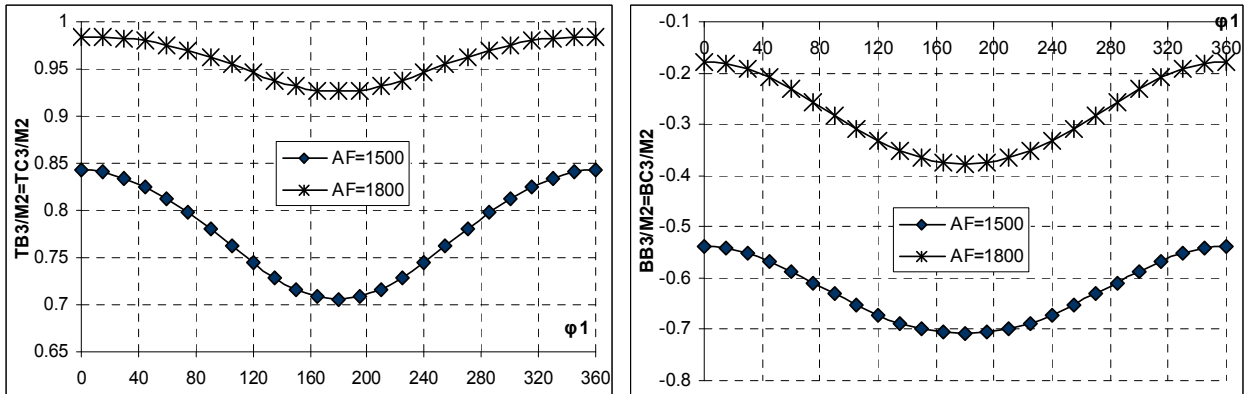


Fig. 12 Variations of the reduced torsion torque $T_{B3}/M_2 = T_{C3}/M_2$, and of the bending moment $B_{B3}/M_2 = B_{C3}/M_2$ for $\alpha = 5^\circ$ and $AF=1500; 1800$ [mm], in respect to φ_1 [deg]

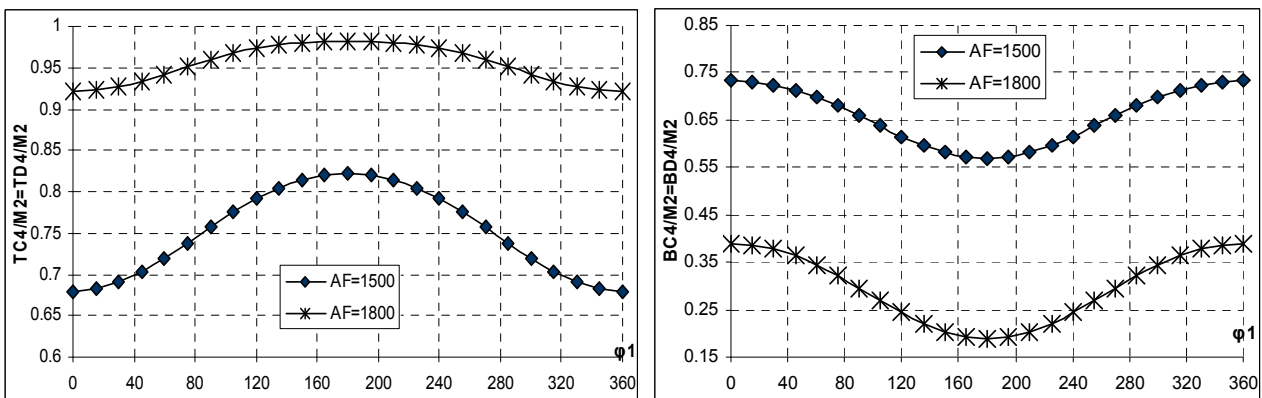


Fig. 13 Variations of the reduced torsion torque $T_{C4}/M_2 = T_{D4}/M_2$, and of the bending moment $B_{C4}/M_2 = B_{D4}/M_2$ for $\alpha = 5^\circ$ and $AF=1500; 1800$ [mm], in respect to φ_1 [deg]

4. CONCLUSIONS

The previous numerical simulations, based on using of the *Excel* software, gave the variations of the torsional and bending torques from the homokinematical eccentric cardan transmission of 8R type (see Fig.8...13). These simulations emphasized:

- the variations of *reduced* torsion torques T/M_2 and *reduced* bending torques B/M_2 , taking the angular displacements from the revolute joints into account, with respect to the angle φ_1 ;
- the influence of angle α (see Fig.1) on the previous *reduced* torques' variations;
- the influence of length AF (see Fig.1) on the previous *reduced* torques' variations.

From the previous graphic variations (see Fig. 8...13), the following conclusions result:

- The amplitude of oscillation of the *reduced* torsional and bending torques increase together with the angle α ;
- The *reduced* torsion torques T_B/M_2 and T_C/M_2 have a variation period of 360° ;
- Because of the transmission's symmetry, the variations of the *reduced* torsional and bending torques (T_B/M_2 , T_C/M_2 , B_B/M_2 , B_C/M_2) from the first part of the transmission are equal with variations of the symmetrical *reduced* torques (T_D/M_2 , T_E/M_2 , B_D/M_2 , B_E/M_2) from the second part;
- The length AF has an significant influence; this influence on the variations of the *reduced* torsional and bending torques decreases with the increase of the length AF (see Fig. 12...13);
- Because of its symmetry, the transmission is homokinematical; according to the previous simulations, a dimensional configuration of the 8R transmission may be optimal (with minimal dynamic off-balance and the minimal variations of the *reduced* torsional and bending torques) for the following data (see Fig.1): the angle $\alpha = 5^\circ$, $AB=EF=150\text{ mm}$, $BC=DE=400\text{ mm}$, $CD=766,281\text{ mm}$, $MM_1 = CD/5$, $AF=1800$.

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